

# **$\tan \beta$ enhanced contributions to $b \rightarrow s + \gamma$ in SUSY without $R$ -parity**

Otto C. W. Kong and Rishikesh Vaidya<sup>a</sup>

*Physics Department, National Central University*

*Chung-Li, 32054, Taiwan*

## ABSTRACT

We present a systematic analysis of the decay  $b \rightarrow s\gamma$  at the leading log within the framework of Supersymmetry without  $R$ -parity. We point out some new contributions in the form of bilinear-trilinear combination of  $R$ -parity violating (RPV) couplings that are enhanced by large  $\tan \beta$ . We also improve by a few orders of magnitude, bounds on several combinations of RPV parameters.

## 1. Introduction

The very existence of a dedicated annual conference on supersymmetry provides ample proof of inadequacy of standard model (SM) as a complete theory, and the appeal of supersymmetry as a most popular candidate for the physics beyond SM. In our opinion, the minimal supersymmetry standard model with conserved  $R$ -parity, lacks the much needed solution to neutrino mass problem which is naturally addressed in models with  $R$ -parity violation (RPV). However, the large number of *a priori* arbitrary RPV couplings must be constrained from phenomenology in all possible ways. In this talk we shall discuss the influence of RPV on the decay channel  $B \rightarrow X_s + \gamma$ . Being loop mediated rare decay, it is sensitive to physics beyond SM. It has already been well measured by CLEO, BELLE, ALEPH and BABAR and hence can be used to put upper bounds on RPV couplings. The experimental world average [1] is  $Br[B \rightarrow X_s + \gamma (E_\gamma > 1.6\text{GeV})]_{\text{SM}} = (3.57 \pm 0.30) \times 10^{-4}$ . Within  $1\sigma$  this matches very well with the SM prediction  $Br[B \rightarrow X_s + \gamma (E_\gamma > 1.6\text{GeV})]_{\text{SM}} = (3.57 \pm 0.30) \times 10^{-4}$  given in [2]. The good agreement between SM prediction and the experimental number at  $1\sigma$  can be used to constrain the large number of *a priori* arbitrary parameters of SUSY without  $R$ -parity.

There have been some studies on the process within the general framework of  $R$ -parity violation. More systematic analysis are exemplified by refs.[3,4]. Ref.[3], fails to consider the additional 18 four-quark operators which, in fact, give the dominant contribution in most of the cases. The more recent work of ref.[4] has considered a complete operator basis. However, we find their formula for Wilson coefficient (WC) incomplete, and they do not report on the possibility of a few orders of magnitude improvement on the bounds for certain combinations of RPV couplings, as we present here [5]. In fact, the particular type of contributions — namely, the one from a combination of a bilinear and a trilinear  $R$ -parity violating (RPV) parameters, we focused on [6], has not been studied in any detail before. Here we shall briefly report the results. For the analytical details we refer the readers to [5,6].

---

<sup>a</sup>Speaker at the conference.

We adopt an optimal phenomenological parametrization of the full model Lagrangian – the single single-*vev* parametrization. It is essentially about choosing a basis for Higgs and lepton superfields in which all the “sneutrino” *vev* vanish. The details and the merits of the parametrization have been discussed at length in [7], and its efficient application for the case of quark dipole-moment and  $\mu \rightarrow e\gamma$  see the references in [5].

## 2. Formalism

The partonic transition  $b \rightarrow s + \gamma$  is described by the magnetic penguin diagram. Under the effective Hamiltonian approach, the corresponding WC of the standard  $\mathcal{Q}_7$  operator has many RPV contributions at the scale  $M_W$ . For example, we separate the contributions from different type of diagrams as  $C_7 = C_7^W + C_7^{\tilde{g}} + C_7^{\chi^-} + C_7^{\chi^0} + C_7^{\phi^-} + C_7^{\phi^0}$  corresponding to W-boson, gluino, chargino, neutralino, colorless charged-scalar and colorless neutral-scalar loops (for details please see [5]). Apart from the 8 SM operators with additional contributions, we actually have to consider many more operators with admissible nonzero WC coefficients at  $M_W$  resulting from the RPV couplings. These are the chirality-flip counterparts  $\tilde{\mathcal{Q}}_7$  and  $\tilde{\mathcal{Q}}_8$  of the standard (chromo)magnetic penguins  $\mathcal{Q}_7$  and  $\mathcal{Q}_8$ , and a whole list of 18 new relevant four-quark operators. For the lack of space, we list 8 important operators below.

$$\begin{aligned} \mathcal{Q}_{9-11} &= (\bar{s}_{L\alpha} \gamma^\mu b_{L\beta}) (\bar{q}_{R\beta} \gamma_\mu q_{R\alpha}) , \quad q = d, s, b; \\ \tilde{\mathcal{Q}}_{9-13} &= (\bar{s}_{R\alpha} \gamma^\mu b_{R\beta}) (\bar{q}_{L\beta} \gamma_\mu q_{L\alpha}) , \quad q = d, s, b, u, c; \end{aligned} \quad (1)$$

and six more operators from  $\lambda''$  couplings[5]. The interplay among the full set of 28 operators is what makes the analysis complicated. The effect of the QCD corrections proved to be very significant even for the RPV parts.

After the QCD running of WC from scale  $M_W$  to  $m_b$ , dictated by  $28 \times 28$  anomalous dimension matrix, the effective WC are given as (at leading log order) [5] :

$$\begin{aligned} C_7^{\text{eff}}(m_b) &= -0.351 C_2^{\text{eff}}(M_W) + 0.665 C_7^{\text{eff}}(M_W) + 0.093 C_8^{\text{eff}}(M_W) - 0.198 C_9^{\text{eff}}(M_W) \\ &\quad - 0.198 C_{10}^{\text{eff}}(M_W) - 0.178 C_{11}^{\text{eff}}(M_W) , \\ \tilde{C}_7^{\text{eff}}(m_b) &= 0.381 \tilde{C}_1^{\text{eff}}(M_W) + 0.665 \tilde{C}_7^{\text{eff}}(M_W) + 0.093 \tilde{C}_8^{\text{eff}}(M_W) - 0.198 \tilde{C}_9^{\text{eff}}(M_W) \\ &\quad - 0.198 \tilde{C}_{10}^{\text{eff}}(M_W) - 0.178 \tilde{C}_{11}^{\text{eff}}(M_W) + 0.510 \tilde{C}_{12}^{\text{eff}}(M_W) + 0.510 \tilde{C}_{13}^{\text{eff}}(M_W) \\ &\quad + 0.381 \tilde{C}_{14}^{\text{eff}}(M_W) - 0.213 \tilde{C}_{16}^{\text{eff}}(M_W) . \end{aligned} \quad (2)$$

The branching fraction for  $Br(b \rightarrow s + \gamma)$  is expressed through the semi-leptonic decay  $b \rightarrow u|ce\bar{\nu}$  (so that the large bottom mass dependence ( $\sim m_b^5$ ) and uncertainties in CKM elements cancel out) with  $Br_{\text{exp}}(b \rightarrow u|ce\bar{\nu}_e) = 10.5\%$  and  $\Gamma(b \rightarrow s\gamma) \propto (|C_7^{\text{eff}}(\mu_b)|^2 + |\tilde{C}_7^{\text{eff}}(\mu_b)|^2)$ . Note that we have also to include RPV contributions to the semi-leptonic rate for consistency[5].

## 3. Results: Impact of bilinear-trilinear combination of parameters

*Analytical Appraisal.*—We implement our (1-loop) calculations using mass eigenstate

expressions[5], hence free from the commonly adopted mass-insertion approximation. While a trilinear RPV parameter gives a vertex, a bilinear parameter now contributes only through mass mixing matrix elements characterizing the effective couplings of the mass eigenstate running inside the loop. The  $\mu_i$ 's are involved in fermion, as well as scalar mixings. There are also the corresponding soft bilinear  $B_i$  parameters involved only in scalar mixings[7]. Combinations of  $\mu_i$ 's and  $B_i$ 's with the trilinear  $\lambda'_{ijk}$  parameters are our major focus.

There are two kinds of  $B_i$ - $\lambda'$  combinations that contribute to  $b \rightarrow s + \gamma$  at 1-loop: (a)  $B_i^* \lambda'_{ij2}$ , and (b)  $B_i \lambda'_{ij3}^*$ . These involve quark-scalar loop diagrams. Case (a) leads to the  $b_L \rightarrow s_R$  transition (where SM and MSSM contribution is extremely suppressed) whereas case (b) leads to SM-like  $b_R \rightarrow s_L$  transition. For the purpose of illustration, we will assume a degenerate slepton spectrum and take the leptonic index  $i = 3$  as a representative. For the  $j$  values, the charged loop contributions are still possible by invoking CKM mixings. Consider the contribution of case (a) with  $|B_3^* \lambda'_{332}|$  to the  $\tilde{C}_7$ , for instance. Through the extraction of the bilinear mass mixing effect under a perturbative diagonalization of the mass matrices[7], we obtain[6],

$$\begin{aligned}\tilde{C}_7^{\phi^-} &\approx \frac{-|V_{\text{CKM}}^{tb}|^2 |B_3^* \lambda'_{332}|}{M_s^2} \left\{ y_b \tan \beta [F_2(x_t) + \mathcal{Q}_u F_1(x_t)] + \frac{y_t m_t}{m_b} [F_4(x_t) + \mathcal{Q}_u F_3(x_t)] \right\} \\ \tilde{C}_7^{\phi^0} &\approx \frac{-2\mathcal{Q}_d y_b |B_3^* \lambda'_{332}| \tan \beta}{M_s^2 M_t^2} F_1(x_b)\end{aligned}\quad (3)$$

for the charged and neutral colorless scalar loop, respectively. Here  $x_t$  stands for  $(m_t^2/M_t^2)$  with an obvious replacement for  $x_b$ .  $F_i$  ( $i = 1 - 4$ ) are the well known loop functions (see [5] for expressions). In the above equations, proportionality to  $\tan \beta$  shows the importance of these contributions in the large  $\tan \beta$  limit. The  $M_s^2$ ,  $M_t^2$ ,  $M_b^2$ , are all scalar (slepton/Higgs) mass parameters. The term proportional to  $y_t$  above has chirality flip into the loop. Thinking in terms of the electroweak states, it is easy to appreciate that the loop diagram giving a corresponding term for  $\tilde{C}_7^{\phi^0}$  (*cf.* involving  $\tilde{\mathcal{N}}_{nm3}^L \tilde{\mathcal{N}}_{nm2}^{R*}$ ) requires a Majorana-like scalar mass insertion, which has to arrive from other RPV couplings[7]. In the limit of perfect mass degeneracy between the scalar and pseudoscalar part (with no mixing) of multiplet, it vanishes. Dropping this smaller contribution, together with the difference among the Inami-Lim loop functions and the fact that the charged loop has more places to attach the photon (with also larger charge values) adding up, we expect the  $\tilde{C}_7^{\phi^-}$  to be larger than  $\tilde{C}_7^{\phi^0}$ . We corroborate these features in our numerical study.

*Numerical Results.*— We take non-vanishing values for relevant combinations of a bilinear and a trilinear RPV parameters one at a time, and stick to real values only. Our model choice for parameters is (with all mass dimensions given in GeV): squark masses 300, down-type Higgs mass 300,  $\mu_0 = -300$  sleptons mass 150 and gaugino mass  $M_2 = 200$  (with  $M_1 = 0.5M_2$  and  $M_3 = 3.5M_2$ ),  $\tan \beta = 37$  and  $A$  parameter 300. We impose the experimental number to obtain bounds for each combination of RPV parameters independently (given in Table I). Consider, for instance, the case (b) combination  $|B_3 \lambda'_{323}^*|$ . We obtain a bound of  $5.0 \times 10^{-5}$ , when normalized by a factor of  $\mu_0^2$ . Since this is a  $b_R \rightarrow s_L$  transition, the RPV contribution interferes with the SM as well as the MSSM

Table 1: **Bounds for the products of bilinear and trilinear RPV couplings.**

Product	Our bound	Product	Our bound	Product	Our bound
$\frac{B_i \cdot \lambda'_{i23}}{\mu_0^2}$	$5.0 \times 10^{-5}$	$\frac{B_i \cdot \lambda'_{i12}}{\mu_0^2}$	$4.5 \times 10^{-2}$	$\frac{\mu_i \cdot \lambda'_{i23}}{\mu_0}$	$2.2 \times 10^{-3}$
$\frac{B_i \cdot \lambda'_{i32}}{\mu_0^2}$	$7.4 \times 10^{-3}$	$\frac{B_i \cdot \lambda'_{i22}}{\mu_0^2}$	$6.5 \times 10^{-2}$	$\frac{\mu_i \cdot \lambda'_{i32}}{\mu_0}$	$1.0 \times 10^{-2}$
$\frac{B_i \cdot \lambda'_{i33}}{\mu_0^2}$	$2.3 \times 10^{-3}$	$\frac{B_i \cdot \lambda'_{i13}}{\mu_0^2}$	$8.0 \times 10^{-2}$	$\frac{\mu_i \cdot \lambda'_{i33}}{\mu_0}$	$8.0 \times 10^{-2}$

contribution. Over and above the loop contributions there are contributions coming from four-quark operator with  $C_{11}$  ( $\propto y_b$ ) which is stronger than the other two four-quark quark coefficients  $\tilde{C}_{10,13} \propto y_s$ . Since the neutral scalar loop contribution is proportional to the loop function  $F_1$  (which is of order .01), it is suppressed compared to current-current contributions. Also here the charged scalar contribution comes only with chirality flip inside the loop and has a CKM suppression. So the current-current is dominant. It has a more subtle role to play when one writes the regularization scheme-independent  $C_7^{\text{eff}} = C_7 - C_{11}$  at scale  $M_W$  (see [5] for details). Due to dominant and negative sign chargino contribution (because  $A_t \mu_0 < 0$ ), the positive sign  $C_{11}$  interferes constructively with  $C_7$  and enhances the rate. These features can be verified from Fig.1 of Ref.[6]. We have done the similar analytical and numerical exercise for all possible combinations of bilinear and trilinear couplings and quote the relevant bounds obtained for the first time in Table 1.

*Conclusions.* — To conclude we have systematically studied the influence of the combination of bilinear-trilinear RPV parameters on the decay  $b \rightarrow s + \gamma$  analytically as well as numerically. These contributions are enhanced by large  $\tan\beta$ . We also demonstrate the importance of QCD corrections and obtain strong bounds on several combinations of RPV parameters for the first time. Numerical study has also been performed on combinations of trilinear parameters[5]. We quote here a few exciting bounds under a similar sparticle spectrum. For instance  $|\lambda'_{i33} \cdot \lambda'^*_{i23}|$  for  $i = 2, 3$  should be less than  $1.6 \times 10^{-3}$  to be compared with rescaled existing bound of  $2 \times 10^{-2}$ .

*acknowledgment.*— R.V. would like to thank K.Hagiwara and KEK for the hospitality during his visit to KEK around the conference period, and the National Science Council of Taiawn for support under post-doc grant number NSC 92-2811-M-008-012.

#### 4. References

- [1] C. Jessop, SLAC-PUB-9610.
- [2] A. Buras *et al.* Nucl. Phys. **B631** 219 (2002).
- [3] B. de Carlos and P.L. White Phys. Rev. **D55** 4222 (1997).
- [4] Th. Besmer and A. Steffen Phys. Rev. **63** 055007 (2001).
- [5] O.C.W. Kong and R. Vaidya hep-ph/0403148.
- [6] O.C.W. kong and R. Vaidya hep-ph/0408115.
- [7] O.C.W. Kong, *Int.J.Mod.Phys. A19*, 1863 (2004).